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Online
Numicon 6 Sample

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Strand and Activity Group Number		Activity Group Title
Preparing for Formal Testing	1	Self-assessment and choosing imagery
Preparing for Formal Testing	2	Problem solving strategies
Numbers and the Number System	1	Working with numbers beyond a million and decimals
Calculating	1	Adding and subtracting negative numbers in context, and large numbers
Calculating	2	Multiplying and dividing
		NPC Milestone 1
Measurement	1	Statistics, charts and graphs
Pattern and Algebra	1	Multiples, factors and primes
Numbers and the Number System	2	Fractions
Calculating	3	Estimating, rounding and equivalence
Calculating	4	Column methods for adding and subtracting
Calculating	5	Percentages
		NPC Milestone 2
Geometry	1	2D shapes and angles
		GMS Milestone 1
Calculating	6	Exploring calculations: multi-step non-routine problems and order of operations
Calculating	7	Ratio and proportion
Measurement	2	Areas of 2D shapes
Calculating	8	Converting fractions and decimals
Pattern and Algebra	2	Exploring number sequences and relationships
		NPC Milestone 3
Measurement	3	3D shapes – nets and surface area
		GMS Milestone 2

Strand and Activity Group Number		Activity Group Title
Calculating	9	Written column methods of multiplying
Calculating	10	Introducing long written methods of dividing
Measurement	4	Volume and scaling
Calculating	11	Adding and subtracting with fractions
Calculating	12	Multiplying and dividing fractions
Pattern and Algebra	3	Using algebra to solve problems
		NPC Milestone 4
Geometry	2	Circles
Calculating	13	Solving non-routine problems using all four operations
	3	Transformations in the four quadrants
		GMS Milestone 3
Pattern and	4	Using symbols and letters for variables and unknowns
		NPC Milestone 5
Preparing for Formal Testing	3	Fluency in calculating with whole numbers and decimals
Preparing for Formal Testing	4	Fluency in calculating with fractions and decimals
Preparing for Formal Testing	5	Preparing to do maths in test conditions

NPC and GMS Investigating activity groups

The investigating activities are independent and can be followed in any order. You may choose to use some or all of the topics with your class, according to their interests and the time available.

NPC Investigating	1	Making squares
NPC Investigating	2	What did I do?
NPC Investigating	3	How many ways?
NPC Investigating	4	Decimal patterns
NPC Investigating	5	Which is the best value?
NPC Investigating	6	An enterprise project
GMS Investigating	1	Shape shifting
GMS Investigating	2	Macro maths
GMS Investigating	3	Interesting information

Pattern and Algebra 4: Using symbols and letters for variables and unknowns

Key mathematical ideas Generalizing, Pattern, Algebra, Functions, Inverse, Equivalence, Mathematical thinking and reasoning

Educational context

In this activity group, children continue to explore how to describe general situations and rules mathematically. They are supported to express patterns numerically, e.g. as sequences and functions, and to identify and describe relationships between numbers, e.g. as formulae. This links to children's work in the *Geometry, Measurement and Statistics 6 Teaching Resource Handbook*, Measurement 2. This leads into describing general rules which apply in any instance of the same type of situation, and, building on their work in Pattern and Algebra 3, to expressing these rules concisely using algebra, with letters standing for unknown values and variables. For example, in Activity 6 they work out how to describe the commutative property of adding two numbers – the property that the order in which the numbers are added doesn't matter – more succinctly, as $a + b = b + a$.
Connecting with the work of Pattern and Algebra 2, we explore general rules of divisibility for help in finding factors.

Learning opportunities

- To describe a numerical pattern or general relationship in words and algebraically, as a formula.
- To recall and use tests of divisibility by 2, 3, 5, 9 and 10.
- To describe and explain the commutative property of adding and multiplying.

Terms for children to use

algebra, algebraic notation, symbol, generalize, reasoning, logic, systematic, show, prove, pattern, sequence, constant difference, term, first term, term-to-term rule, predict, relationship, general rule, general term, *n*th term, unknown, variable, value, expression, equation, equivalent, inverse, function, function machine, input, output, divisibility, test of divisibility, factor, multiple, prime, composite, commutative property, associative property, number trio, part–whole relationship

Assessment opportunities

Look and listen for children who:
- Use the terms for children to use effectively.
- Can identify the term-to-term rule in a linear sequence, e.g. in the sequence 38, 43, 48, 53, … the term-to-term rule is 'add 5'.
- Describe a rule for finding the general term of a linear sequence and express this with an algebraic expression, e.g. $5n + 33$ in Activity 1.
- Can explain algebraically how 'think of a number' problems work.
- Can explain the general relationship between an 'input' (x) and an 'output' (y) for a particular function (e.g. for a function described by $y = 3x$, y is always three times x, x is always one third of y).
- Can identify a missing input or output for a given function machine, and a missing instruction, e.g. '× 3' for a given set of inputs and outputs.
- Can write an equation to show the general relationship between input and output for a given function, represented as x and y respectively, e.g. $y = 3x$.
- Use tests of divisibility to sort numbers.
- Describe the commutative properties of adding and of multiplying in general terms, including algebraically, e.g. $a + b = b + a$, $ab = ba$.
- Can explain why adding and multiplying are commutative, while subtracting and dividing are not.

NPC Milestone 5

- Use symbols and letters to represent variables and unknowns in mathematical situations (NPC6:5a)

Explorer Progress Book 6b, pp. 20–23

After completing work on this activity group, give small focus groups of children their Explorer Progress Books and ask them to work through the challenges on the pages. As children complete the pages, assess what progress they are making with the central ideas from the activity group. Refer to the assessment opportunities for assistance.
Children will also have the opportunity to complete their Learning Log (pp. 22–23) where they can reflect on the mathematics they have done.

Explore More Copymaster 4: Secret Function Machine

After completing work on Activity 4, give children Explore More Copymaster 4: Secret Function Machine to take home.

Pupil Book 6, pp. 102–105

These pages in the Pupil Book provide further practice and challenging questions. You can use them to follow up the activities and deepen the learning.

Focus activities

1. Investigating rules and generalizing with algebra
2. Generalizing about linear sequences using symbols and letters
3. Generalizing about 'think of a number' problems
4. Using symbols to describe function machines
5. Generalizing about divisibility
6. Expressing general laws of arithmetic

Key Mathematical Ideas provide a summary of the important concepts covered this week

Learning Opportunities are linked with the Assessment opportunities, detailing the range of Focus Activities for this week

Assessment is supported by Explorer Progress activities at the end of the week or later. These are recorded along with the Milestones to provide a record of learning that is stored in the assessment Tracker

I: Investigating rules and generalizing with algebra - part 1

Quit activity

Intro | Links | 1 | 2 | 3 | 4 | 5 | 6 | 7

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Learning opportunities:

- See all learning opportunities

Terms for children to use:

algebra, algebraic notation, symbol, generalize, reasoning, logic, systematic, show, prove, pattern, sequence, constant difference, term, first term, term-to-term rule, predict, relationship, general rule, general term, *n*th term, unknown, variable, value, expression, equation, equivalent, inverse, function, function machine, input, output, divisibility, test of divisibility, factor, multiple, prime, composite, commutative property, associative property, number trio, part–whole relationship

Have ready:

- Numicon Shapes
- 100 square or 1000 square on the Numicon IWB Software
- number rods

I: Investigating rules and generalizing with algebra - part 1

Quit activity

Intro | Links | 1 | 2 | 3 | 4 | 5 | 6 | 7

Links:

Full activity group overview
Starter image
Whole-class practice and discussion
Photocopy masters
Implementation Guide
Pupil Book 6 opening questions p. 102
Pupil Book 6 Answer Book
IWB Software
MyMaths

Next steps:

Explorer Progress Book 6b, pp. 20–23
Explore More Copymaster 4: Secret Function Machine
Numicon 6 Milestone Assessment cards (NPC 6:5a)
Numicon 6 Milestone Tracking chart

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Implementation Guide

Using symbols and letters for variables and unknowns

For many people, doing 'algebra' means 'using letters instead of numbers', and there is some rough sense in this. There are occasions when we want to talk about either a range of numbers (and not one particular number), or about a particular value when we don't know what it is. In both situations, since (for these two quite different reasons) we can't specify particular numbers, we use letters to talk about relationships between numbers instead.

In the *Number, Pattern and Calculating 6 Teaching Resource Handbook*, children progress from finding unknowns in single 'empty box' number sentences such as □ – 24 = 37, to finding pairs of numbers that satisfy number sentences involving two unknowns. Children will need to think systematically, to explore the possible pairs of numbers that will satisfy number sentences such as r + □ = 7, or 12 – r = □. It is important to vary the operations involved as well, and to explore number pairs that satisfy sentences such as □ × 8 = r, or even a ÷ b = 12.

An increasingly important aspect of work in Number, Pattern and Calculating 6 involves children learning to use letters to express relationships between numbers and unknown quantities of various kinds symbolically. This aspect of children's mathematical communicating – and hence their mathematical thinking – develops significantly at this stage, particularly in relation to the idea of a function.

Functions – a special kind of pattern in mathematics: In Number, Pattern and Calculating 6 children continue to work on particular patterns that are called 'functions'. Mathematical functions are used to handle relationships of dependence between changing values, for example, the relationship between time, speed, and distance travelled as we move. The distance travelled at any point depends upon how fast we have been going and how long we have been travelling.

Central to the general idea of a function is that of a 'variable'; this is what mathematicians call

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Practice and discussion: Whole-class

- Discuss with children how and when the mathematics they have been learning could help them in solving problems.
- Talk with children about relationships and formulas they have used, e.g. the area of a rectangle is given by the product of its length and width, its perimeter is double the sum of its length and width. Work with them to write these algebraically, e.g. area = lw, perimeter = $2(l + w)$.
- Show a group of Numicon Coloured Counters (see example), explaining that each different colour of Counter stands for a different number. Support children to write an algebraic expression for the total value of the Counters, e.g. $3b + 4r$, then to substitute into the expression to find the total value if, e.g. each blue Counter is worth 32 and each red Counter is worth 26.

- Give algebraic expressions, e.g. $3q$, $3n + 2$, and values for the variables, e.g. $q = 40$, $n = 3$, for children to find the value of the expression.
- Give a letter to represent a variable, e.g. x. Ask children to write the expression for the number which is, e.g. 3 times as big ($3x$), one-tenth as big (e.g. $0{\cdot}1x$, $x \div 10$, $\frac{x}{10}$), 12 less ($x - 12$).
- Work with children to make a pattern with, e.g. Numicon Coloured Counters, construction sticks or interlocking cubes, then describe the resulting sequence and a general rule for finding any term.
- Provide a linear sequence, e.g. 4, 9, 14, 19, ... , for children to illustrate with Numicon Shapes or number rods. Work with children to describe a general rule for finding any given term and identify an expression for the *n*th term, e.g. $5n - 1$.
- Draw function machines for children to identify the missing number in each one.
- Show number sentences relating x to y, e.g. $y = x - 4$, $y = 0{\cdot}25x$, $y = 2x + 1$, for children to draw the related function machines.
- Give children 3-, 4- and 5-digit numbers to sort according to divisibility, e.g. into 'multiples of 9' and 'not multiples of 9'.

Focus activities

1. Investigating rules and generalizing with algebra
2. Generalizing about linear sequences using symbols and letters
3. Generalizing about 'think of a number' problems
4. Using symbols to describe function machines
5. Generalizing about divisibility
6. Expressing general laws of arithmetic

Step 1

Show a 100 square and choose a 'starting number', e.g. 5. Give a starting rule, e.g. 'find the total of the starting number, the two numbers to its right and the two numbers below it,' illustrating on the 100 square (see image).

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

Agree that, following the rule, we get $5 + 6 + 7 + 15 + 25 = 58$.

Step 2

Ask children to investigate the result of using different starting numbers. Support them to work systematically and organize their findings (see image).

(When children get to the last two starting numbers in a row, e.g. 9 and 10 in the first row, guide them to interpret reading 'to the right' as reading on to the next line of the 100 square where necessary, e.g. so that the two numbers 'to the right' of 9 are 10 and 11.

Similarly, if they use starting numbers greater than 80, guide them to visualize the 100 square continuing so that, e.g. the two numbers 'below' 81 are 91 and 101. That is, use the 100 square only as a way of starting off the sequence, then encourage children to identify the pattern in the numbers and apply it to generate further terms.)

Step 3

Encourage children, as they work, to illustrate their findings with apparatus or imagery of their choice and to look for patterns and relationships among both the numbers being added and the totals.

Look and listen for those who spot the repeating pattern in the ones digit of the totals (8 where the starting number is odd, 3 where it is even) and the constant difference between each total and the next (5).

Agree that the totals form a sequence with first term 38 and term-to-term rule 'add 5' (make links, as needed, to children's previous work on sequences, e.g. in Pattern and Algebra 2 or in the *Number, Pattern and Calculating 5 Teaching Resource Handbook*, Pattern and Algebra 1).

Step 4

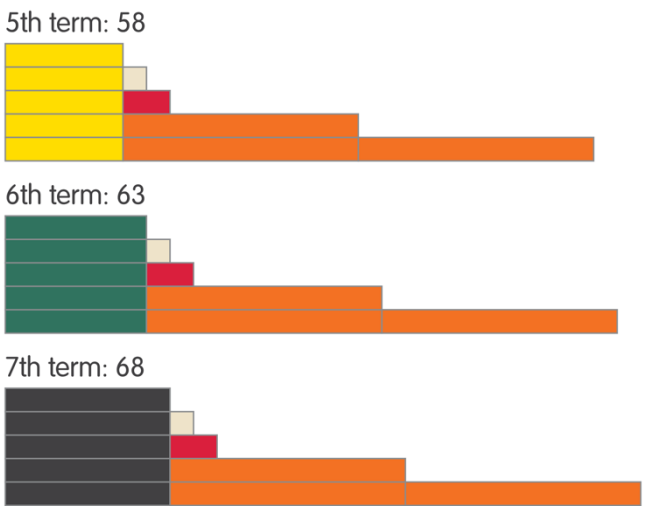
Ask children whether there is a way to predict the 22nd term in the sequence (without simply repeatedly adding 5). Some may suggest using patterns, e.g. identifying that the ones digit for the 22nd term will be 3, since 22 is an even number. Encourage them instead to look for relationships between the number of the term and the numbers being totalled, allowing plenty of time for them to experiment and explore.

Step 5

Look and listen for children who can illustrate and describe in general terms the relationships between the numbers which are added together to give each term in the sequence. Invite them to explain their thinking.

Agree that the starting number always matches the position number of the term, so, e.g. the 5th term has starting number 5. Guide children to explain that the two numbers to the right of the starting number are always 1 more and 2 more than the starting number; then, similarly, the two numbers below are always 10 more and 20 more than the starting number. Encourage children to come up with ways to illustrate this visually (see image).

Invite children to use their illustration to explain why the term-to-term rule for the sequence is 'add 5'. Look and listen for children who can explain, e.g. the only number that changes when adding to find a new term is the starting number, which is 1 more each time; since it is added into the total 5 times, each term is $1 \times 5 = 5$ more than the previous term.



Step 6

Work with children to write number sentences to show how to use these relationships to calculate some of the terms, e.g.

5th term:
5 + (5 + 1) + (5 + 2) + (5 + 10) + (5 + 20) = 58

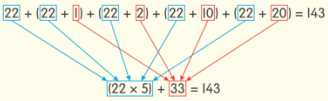
6th term:
6 + (6 + 1) + (6 + 2) + (6 + 10) + (6 + 20) = 63

7th term:
7 + (7 + 1) + (7 + 2) + (7 + 10) + (7 + 20) = 68

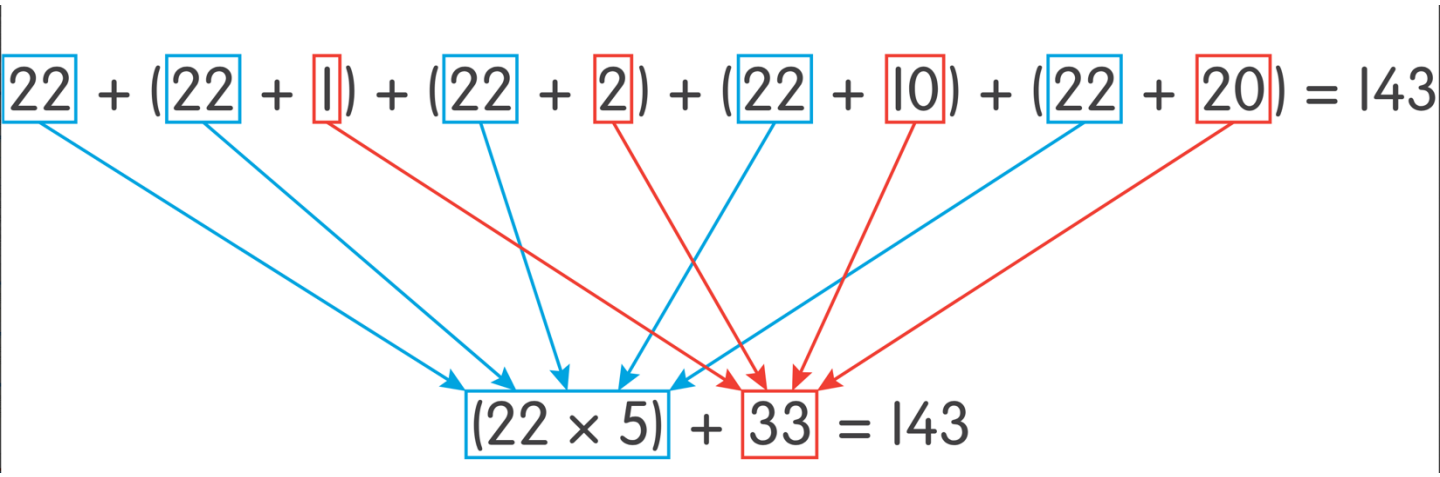
Prompt children, as needed, to generalize to describe in their own words a rule for finding any term, e.g. 'Add 5 lots of the position number, then add 33.'

Step 7

Talk with children about how to calculate the 22nd term. Work with them to use the pattern identified in Step 6, replacing the starting number with the position number of the term; then use the general rule identified in Step 6, pointing out links between the two calculations (see image). Agree that the 22nd term is 143.



I: Investigating rules and generalizing with algebra - part 2

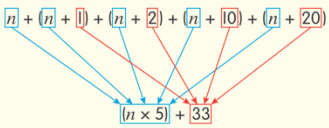


Step 8a

Recall with children their work on using letters and symbols to represent unknowns or variables (e.g. in Pattern and Algebra 3). Tell children that *n* is often used to represent any number in a sequence; that is, *n* represents a 'general' term of a sequence.

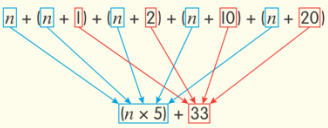
Ask children whether they can write an expression for a term with starting number *n*. Once they have had time to talk about and explore this, invite them to share their ideas.

Work with them to use the pattern identified in Step 6, then the general rule, making links between the two calculations (see image).



Step 8b

Invite children to confirm that this expression $(n \times 5) + 33$ or $5n + 33$ is correct by substituting values for *n* in order to calculate terms they have found previously: the first term is $(1 \times 5) + 33 = 38$, the second term is $(2 \times 5) + 33 = 43$, and so on (see image).

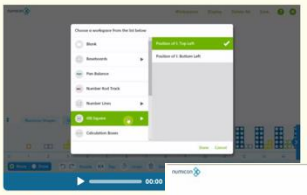


Step 9

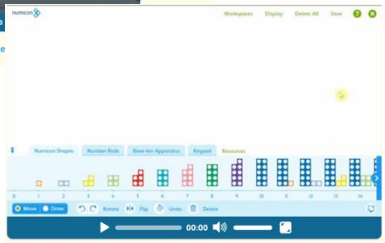
Work with children to use a different rule, e.g. 'find the total of the starting number, the number on each side and the two numbers diagonally to top left and bottom right' (see image 1) – limit the starting numbers to 2 digits and wrap around as in Step 2), to find the first few terms of a sequence, then an expression using *n* as the starting number (see image 2). (For the example given here, use, e.g. 12 as the first starting number.) Look and listen for children who can explain the patterns in the numbers being added and the link to the expression using *n*.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

Starting number	Calculation	Result
12	12 + 11 + 13 + 1 + 23	60
13	13 + 12 + 14 + 2 + 24	65
14	14 + 13 + 15 + 3 + 25	70
n	$n + (n - 1) + (n + 1) + (n - 11) + (n + 11)$	$5n$



Notes for Investigating rules and ge



Notes for Investigating rules and generalizing with algebra Part 2

I: Investigating rules and generalizing with algebra - part 2

Quit activity

Intro | Links | 8a | 8b | 9 | Video | Video | +

Paired or individual work

Have ready: 100 square

Give children the opportunity to investigate further rules for generating sequences using a 100 square, e.g. 'Find the total of a starting number and all eight numbers surrounding it.' Challenge them to explain the patterns in the numbers being added and write an expression for a general term with starting number *n*.

(For the example given here, 9*n*; prompt children to choose starting numbers which are easy to work with, e.g. 12. Remind children that the 100 square is only used to start the sequence; thereafter, they should use the term-to-term rule.)

Numicon Pupil Book 6 pp. 102–105
Numicon Pupil Book 6 Answer Book p. 43

Pupil Book 6 opening questions - part 3

Quit activity

p.74 | p.78 | p.82 | p.86 | p.90 | p.94 | p.98 | p.102

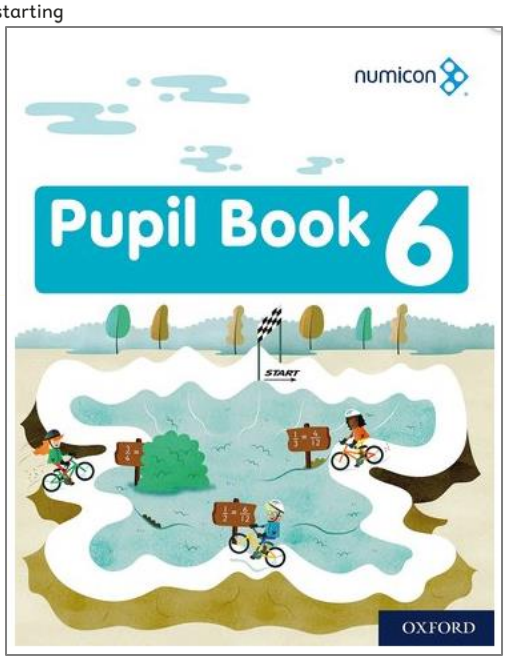
Pattern and Algebra 4·1

Using symbols and letters

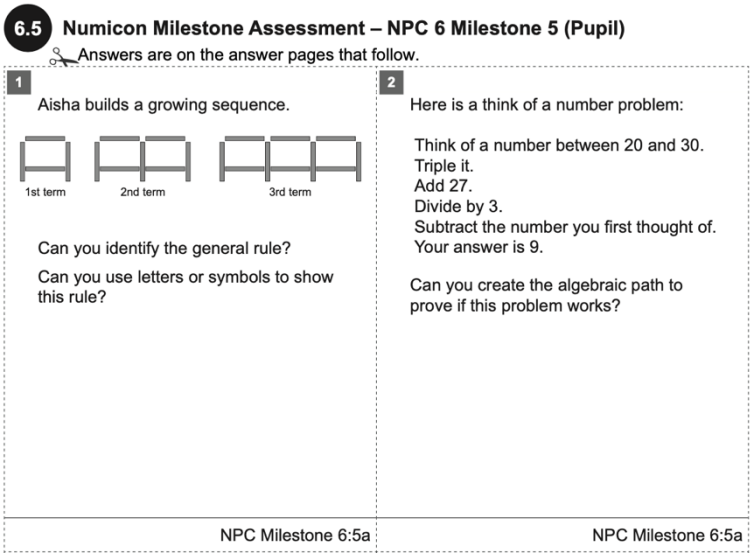
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

Practice

1 The number pattern above used 26 as its starting number. Can you describe the rules that connect the other two numbers to the starting number?



Milestone
ASSESSMENT CARDS



6.5 Numicon Milestone Assessment – NPC 6 Milestone 5 (Pupil)

Answers are on the answer pages that follow.

1

Aisha builds a growing sequence.

Can you identify the general rule?

Can you use letters or symbols to show this rule?

NPC Milestone 6:5a

2

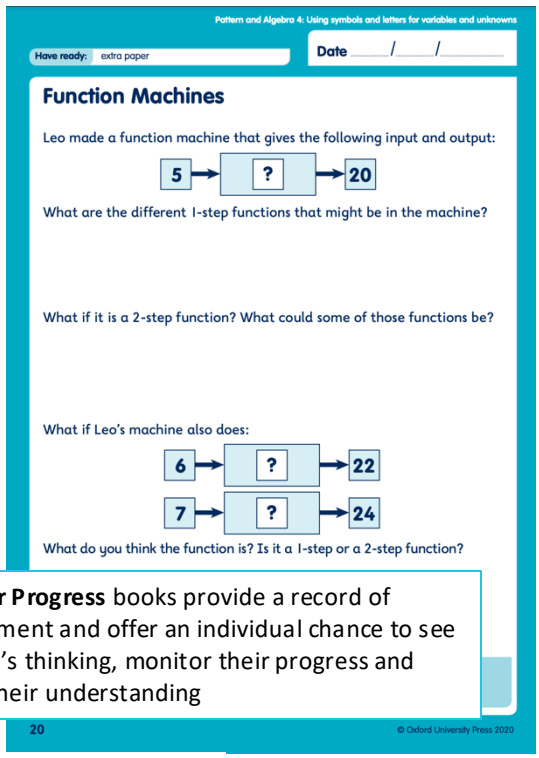
Here is a think of a number problem:

Think of a number between 20 and 30.
Triple it.
Add 27.
Divide by 3.
Subtract the number you first thought of.
Your answer is 9.

Can you create the algebraic path to prove if this problem works?

NPC Milestone 6:5a

Explorer Progress
– weekly assessment tool



Pattern and Algebra 4: Using symbols and letters for variables and unknowns

Have ready: extra paper

Date ..../..../....

Function Machines

Leo made a function machine that gives the following input and output:

5 → ? → 20

What are the different 1-step functions that might be in the machine?

What if it is a 2-step function? What could some of those functions be?

What if Leo's machine also does:

6 → ? → 22
7 → ? → 24

What do you think the function is? Is it a 1-step or a 2-step function?

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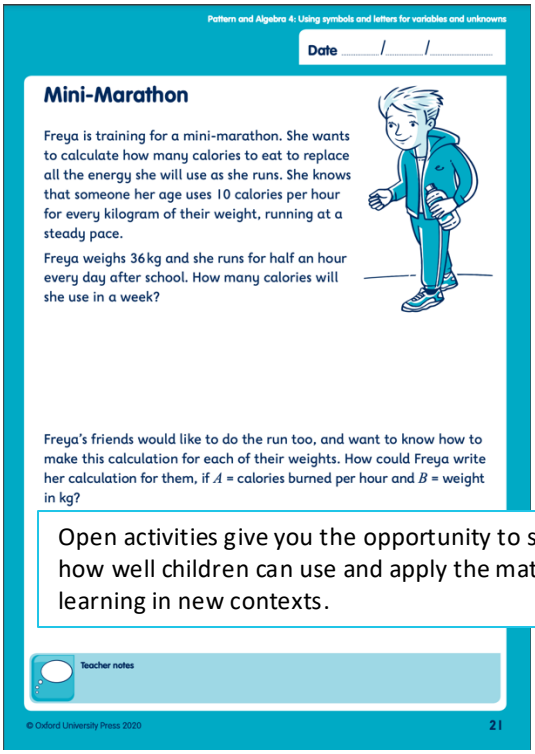
Explorer Progress books provide a record of achievement and offer an individual chance to see children's thinking, monitor their progress and assess their understanding



Pattern and Algebra 4: Using symbols and letters for variables and unknowns

Date ..../..../....

Mini-Marathon

Freya is training for a mini-marathon. She wants to calculate how many calories to eat to replace all the energy she will use as she runs. She knows that someone her age uses 10 calories per hour for every kilogram of their weight, running at a steady pace.

Freya weighs 36 kg and she runs for half an hour every day after school. How many calories will she use in a week?

Freya's friends would like to do the run too, and want to know how to make this calculation for each of their weights. How could Freya write her calculation for them, if A = calories burned per hour and B = weight in kg?

Teacher notes

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Open activities give you the opportunity to see how well children can use and apply the maths learning in new contexts.

Assessment Tracker
– weekly assessment tool

Number, Pattern & Calculating 6 Milestone 5						Started	Started	Started	Not s
By this point, children should be able to:						1 out of 2	0 out of 2	0 out of 2	0 ou
• Use symbols and letters to represent variables and unknowns in mathematical situations	NPC6:5a	NPC	P&A	P&A4	Algebra				
• Solve non-routine problems using all four operations	NPC6:5b	NPC	C	C13	Add, subtract, multiply and divide				

GREEN – Achieved
ORANGE – on the way
RED – to target
WHITE – not started

My Learning Log

Date ..../..../....

I know how to...

Date ..../..../....

I know these facts:

Date ..../..../....

I can use maths in everyday life when...

Date ..../..../....

Reflections.

My favourite maths was...

I would like more time to think about...

I felt proud when...

Explorer Progress Books

Activities for class and home offer further opportunities for children to explore maths in an engaging way.

A clear guide and suggestions on how to extend the activity

Secret Function Machine

How this will help your child

- This activity will help your child to look for relationships between numbers.
- It will also help them to express these relationships in words and letters (algebraically).
- It will also allow them to practise finding the 'missing rule' using inputs and outputs of function machines.

Words and phrases to use

relationship, general rule, function, function machine, input, output, squared (multiply a number by itself)

You will need

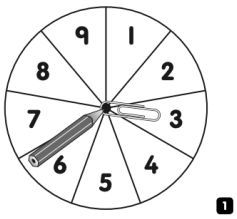
- Scissors
- 2 pencils
- A paper clip
- Card Numicon Shapes 1–10 (optional)
- Card Numicon 10-shapes (optional)

During the activity, look at what your child can do

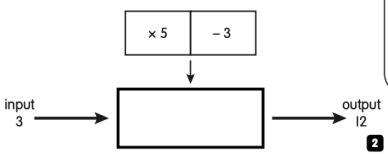
- Identify a missing function to explain how two numbers are related (e.g. the missing function between 10 and 11 might be '+ 1').
- Write an equation to show how two numbers are related.

What to do

- Use a paper clip and pencil to make the spinner from the Secret Function Machine sheet. 1
- Cut out the function cards from the sheet (leave out the blank cards). Place the cards face down in a pile.
- Ask your child to spin the paper clip on the spinner. The number spun is the 'input', e.g. 3.
- Pick up a function card from the pile, without showing your child. Write the input in pencil on the left-hand side of the function machine. 2
- Tell your child the 'output', e.g. if you pick up '× 5, – 3', multiply their number by 5 and subtract 3; so if a 3 is spun, the output is 12 (3 × 5 = 15; 15 – 3 = 12). Record the output in pencil on the right-hand side of the function machine. 2
- Explain to your child that they have to work out the function of the machine (what is written on the card) and that this could involve two instructions. They can use Card Numicon Shapes to help them.
- Ask your child to spin the spinner again. Say the output using the same function. Record the input and output on the function machine.
- Keep going until your child can work out what the secret function is.
- Encourage your child to describe how the input and output are related using x for the input and y for the output, e.g. y is five times x then subtract 3; $y = 5x - 3$.
- Repeat with different function cards.



1



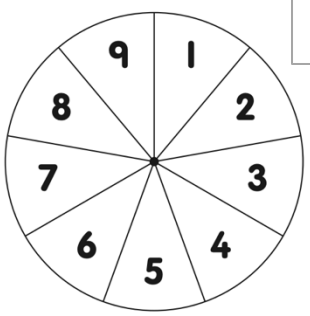
2

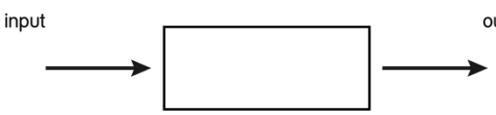
Next steps…

- Make up your own functions using the blank function cards and complete the activity again.
- Ask your child to find the missing function when converting pounds to pence (× 100), kilograms to grams, or kilometres to metres (× 1000).

Name ..... Date .....

Secret Function Machine





× 5	– 3	× 5	+ 6		
squared	– 5	× 4			

Practical real-life contexts help children think about how maths can be used and applied

Your next steps…

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